

M-math 2nd year Supplementary Back Paper Exam
Subject : Analysis III

Time : 2.30 hours

Max.Marks 45.

1. Show that

$$\int_C ydx + zdy + xdz = \pi a^2 \sqrt{3}$$

where C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and the plane $x + y + z = 0$. (10)

2. Let $T = [0, 1] \times [0, 1]$ with coordinates (u, v) and $\vec{r} : T \rightarrow S \subset \mathbb{R}^3$ be a smooth parametrisation of a surface S . Let C^* be a smooth curve in T and let $C := \vec{r}(C^*)$ be a smooth curve lying on S . Show that at each point of C , the vector cross product $\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}$ is normal to C . (10)

3. Let $V_n(a) := \int \cdots \int_{S_n(a)} dx_1 \cdots dx_n$ where $S_n(a) := \{(x_1, \cdots, x_n) : x_1^2 + \cdots + x_n^2 \leq a^2\}$ and the integral is an n -dimensional multiple integral. Show that $V_n(1) = C_n V_{n-2}(1)$, $n \geq 3$ where C_n is some constant depending only on n . (10)

4. Compute the work done by the force field $f(x, y) := (y + 3x, 2y - x)$ in moving a particle once around the ellipse $4x^2 + y^2 = 4$. (10)

5. Compute the volume of the solid enclosed by the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \tag{10}$$